Let’s Be Reasonable
Fostering Fraction Sense

Brian Bushart
Elementary Mathematics Curriculum Coordinator
Beyond Pizzas & Pies
SECOND EDITION
10 Essential Strategies for Supporting Fraction Sense
Julie McNamara
Meghan M. Shaughnessy
Foreword by Francis (Skip) Fennell
Estimate the answer to \( \frac{12}{13} + \frac{7}{8} \).

A 1
B 2
C 19
D 21
E I don’t know

8th Grade, NAEP, 1996
Estimate the answer to $\frac{12}{13} + \frac{7}{8}$.

A 1  7%
B 2  24%
C 19  28%
D 21  27%
E I don’t know  14%

8th Grade, NAEP, 1996
“If I give my students an expression with whole numbers, they’re fine. If I give them the same type of problem, but it includes a fraction, they act like they have no idea what to do.”

-- My friend Meredith, Middle School Teacher, 2010
Which fraction has a value closest to $\frac{1}{2}$?

A. $\frac{5}{8}$

B. $\frac{1}{6}$

C. $\frac{1}{5}$

D. $\frac{2}{2}$

4th Grade, NAEP, 2009
Which fraction has a value closest to $\frac{1}{2}$?

A \[\frac{5}{8}\] 25%

B \[\frac{1}{6}\] 6%

C \[\frac{1}{5}\] 41%

D \[\frac{2}{2}\] 26%

4th Grade, NAEP, 2009
“The difficulty with fractions (including decimals and percents) is pervasive and is a major obstacle to further progress in mathematics...”

Why is understanding fractions so hard?

- Fraction notation - numbers considered in new ways
- Practices that simplify and/or mask the meaning of fractions
- Over reliance on whole number knowledge
- Many meanings of fractions
Adjectives vs Nouns

Young children initially consider whole numbers as **adjectives** or descriptors.

- 9 bears
- 6 cookies
- 20 students
Adjectives vs Nouns

Eventually they come to understand whole numbers as **nouns** or concepts.

- 9 is two more than 7
- 9 is one less than 10
- 9 is 3 squared
- 9 is the square root of 81
Adjectives vs Nouns

Students need opportunities to transition from considering fractions as adjectives...

- $\frac{1}{2}$ of a pizza
- $\frac{2}{3}$ of a cup
- $\frac{3}{4}$ of an hour
Adjectives vs Nouns

...to considering fractions as **nouns**.

- $\frac{5}{8}$ is a little more than $\frac{1}{2}$ but less than 1
- $\frac{5}{8}$ is $\frac{3}{8}$ away from 1
- $\frac{5}{8}$ is half of 1 $\frac{1}{4}$
“It may be surprising that, for most students, to think of a rational number as a number - as an individual entity or a single point on a number line - is a novel idea.”

This Doesn’t Help

The Story of the Cowboy Method

\[
\begin{align*}
\frac{3}{5} & = \frac{0.6}{0.6} \\
& = \frac{3}{0.6} \\
\text{answer: } & = \frac{3}{3} = 0.6
\end{align*}
\]

The cowboy rides the horse. The cowboy goes inside the house, takes off his boots, hangs up his hat, and eats three donuts. The horse stays outside the house.

On Your Own!

\[
\begin{align*}
\frac{1}{8} & = \frac{9}{12} \\
\frac{2}{3} & = \frac{3}{5} \\
\frac{9}{10} & = \frac{16}{5}
\end{align*}
\]
The cowboy rides the horse. The cowboy goes inside the house, takes off his boots, hangs up his hat, and eats three donuts. The horse stays outside the house.
Three Problems With...

Two common approaches to comparing fractions

- Finding a common denominator
- Cross-multiplying
1. Overgeneralizing

\[
\begin{align*}
\frac{4}{8} & \quad \frac{1}{3} & \quad \frac{3}{5} \\
\downarrow & \quad \downarrow & \quad \downarrow \\
\frac{60}{120} & \quad \frac{40}{120} & \quad \frac{72}{120}
\end{align*}
\]
2. Masking

Cross-Multiplication

\[
\begin{align*}
\frac{5}{6} \times \frac{7}{8} &= 5 \times 8 = 40 \\
40 < 42, \text{ so } &\frac{5}{6} < \frac{7}{8}
\end{align*}
\]

\[
\begin{align*}
\frac{5}{6} \times \frac{8}{8} &= \frac{40}{48} \\
\frac{7}{8} \times \frac{6}{6} &= \frac{42}{48} \\
\frac{40}{48} < \frac{42}{48}, \text{ so } &\frac{5}{6} < \frac{7}{8}
\end{align*}
\]
The Third Problem

Two common approaches to comparing fractions

- Finding a common denominator
- Cross-multiplying

These two strategies work for all cases, but they do not explicitly require a consideration of the size of the fractions.
Robbing Students of Sense Making

Both answers look good to students!
What can we do about this?
Some Ideas

- Fraction Applet
- Comparing Fraction Stories
- Number Line Flip
- TEKS Scaffolds and IQ Documents
  - Notice and Wonder
Fraction Applet

[Image of Fraction Applet]

Link
Other Resources

Comparing Fraction Stories

- John’s Pizza
- Lunch With Friends
- Eating Half

John’s Pizza
A Comparing Fractions Story

Brian Stockus
Number Line Flip
# TEKS Scaffold and IQ Documents

## TEKS Scaffold: Math Grade 5

### 5.4
The student applies mathematical process standards to develop concepts of expressions and equations.

<table>
<thead>
<tr>
<th>#</th>
<th>Readiness Standard</th>
<th>Type</th>
<th>Checkpoint 1</th>
<th>Checkpoint 2</th>
<th>Checkpoint 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.4(A)</td>
<td>recognize the difference between additive and multiplicative numerical patterns given in a table or graph</td>
<td>☑️</td>
<td>☑️</td>
<td>☑️</td>
<td></td>
</tr>
<tr>
<td>5.4(B)</td>
<td>describe the process for graphing ordered pairs of numbers in the first quadrant of the coordinate plane</td>
<td>☑️</td>
<td>☑️</td>
<td>☑️</td>
<td></td>
</tr>
<tr>
<td>5.4(C)</td>
<td>describe the key attributes of the coordinate plane, including perpendicular number lines (axes) where the intersection (origin) of the two lines coincides with zero on each number line and the grid point (0, 0); the x-coordinate, the first number in an ordered pair, indicates movement parallel to the x-axis starting at the origin; and the y-coordinate, the second number, indicates movement parallel to the y-axis starting at the origin</td>
<td>☑️</td>
<td>☑️</td>
<td>☑️</td>
<td></td>
</tr>
</tbody>
</table>

### 4.5(B)
represent problems using an input-output table and numerical expressions to generate a number pattern that follows a given rule representing the relationship of the values in the resulting sequence and their position in the sequence.

### 4.5(F)
represent real-world relationships using number pairs in a table and verbal descriptions.

### 2.7(A)
determine whether a number up to 40 is even or odd using pairing of objects to represent the number.

### 2.2(B)
model, create, and describe contextual division situations in which

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@bstockus  @EMathRRISD

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2013-2016 Released Tests
Aligned to the Standards
CONTENT BUILDER FOR THE PLC
To get to a specific section of this document, click on one of the tabs above.

Grade 5
Grade 4
Grade 3
Notice and Wonder
Notice and Wonder

1. Anissa shaded part of the figure shown below.

What fraction of the figure is shaded?

A. $\frac{6}{8}$
B. $\frac{1}{6}$
C. $\frac{2}{8}$
D. $\frac{2}{6}$
Notice and Wonder
Notice and Wonder

Amelia shaded $\frac{2}{8}$ of a rectangle. Which rectangle shows $\frac{2}{8}$ shaded?

A

B

C

D
Notice and Wonder
Notice and Wonder

7  Point Y is labeled on the number line.

Which statement is true?

A  Point Y represents $\frac{3}{6}$ and $\frac{3}{4}$, because both fractions represent 3 equal parts of a whole.

B  Point Y represents $\frac{3}{6}$ and $\frac{1}{2}$, because both fractions are exactly halfway between 0 and 1 on the number line.

C  Point Y represents $\frac{4}{6}$ and $\frac{3}{6}$, because both fractions represent 6 equal parts of a whole.

D  Point Y represents $\frac{4}{6}$ and $\frac{1}{2}$, because both fractions are exactly halfway between 0 and 1 on the number line.
Notice and Wonder
Notice and Wonder
Notice and Wonder

Ever Wonder What They’d Notice?
(if only someone would ask)

Annie Fetter
The Math Forum @ Drexel

Link
Notice and Wonder
“Understanding the relationship between representations of the same idea is key to mathematical understanding.”

-- Beyond Pizzas and Pies, page 116
Notice and Wonder
Notice and Wonder
“Helping students understand the meaning of fractions in different contexts builds their understanding of the relevant features of different fraction representations and the relationships between them.”

-- Beyond Pizzas and Pies, page 117
Which One Doesn’t Belong?
Which One Doesn’t Belong?
Which One Doesn’t Belong?
“Providing opportunities for students to reason about fractional values lays an important foundation for further work involving fractions.”

-- *Beyond Pizzas and Pies*, page 144
Fraction Talks
Fraction Talks
Fraction Talks
Fraction Talks

How many ways can you outline a rectangle that is...

● ½ blue?
● ⅓ blue?

How many rectangles can you find that are ¼ brown?

Can you outline a rectangle that is just as brown as it is blue?
“It is well documented that a deep understanding of equivalent expressions and equalities is a necessary aspect of algebra readiness.”

-- Beyond Pizzas and Pies, page 61
Other Resources

Beyond Pizza & Pies
10 Essential Strategies for Supporting Fraction Sense
Julie McNamara
Meghan M. Shaughnessy
Foreword by Francis (Gus) S. Wurtzel
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Beyond Invert & Multiply
Making Sense of Fraction Computation
Julie McNamara
Foreword by Deborah (Debbie) Schifter

Other Resources

- Counting Circles
- Choral Counting
- Ways to Make a Number
- Today’s Number
- Organic Number Line
Other Resources

Instructional Routines

- Contemplate Then Calculate
- Quick Images
- Choral Counting
- True/False Equations
- Number Strings

 TEDD.ORG
Be in touch.

What did you try?
How did it go?
What can you share?